SYLLABUS

Name of the course: Optimization problems in fluid mechanics
Level of studies: MSc., Ph. D.
Direction, specialization: Mechanical Engineering (KWPI, MOSKOM), Power engineering
Code of the course: **Semester: 2** **ECTS: 4**
Level of the course: advanced **Type**: elective

Total workload:	90 h	Lectures:	36 h		
		Tutorials:	0 h		
		Labs:	0 h	Self-study:	20 h
		Consultations:	4 h		
		Project:	30 h		

Head of the course: prof. dr hab. inż. Jacek Szumbarski

Educational objectives:

C1. Grounding and systematizing knowledge on mathematical formulation of boundary- and initial/boundary value problems of fluid mechanics.
C2. Learning mathematical foundations of optimization and optimal control problems in fluid mechanics.
C3. Learning of theoretical foundations of shape-calculus-based optimization.
C4. Familiarization with selected examples of optimal control of heat and fluid flows in fixed internal domains.
C5. Familiarization with selected examples of static and dynamic shape optimization.
C6. Development of competences in numerical analysis of complex optimization problems via realization of a team project.

Assumed knowledge and skills

1. Fluid mechanics – 1st level of engineering studies.
2. General optimization – 1st level of engineering studies.
3. Basic knowledge and skills in numerical methods applied to initial and boundary value problems for ODEs (finite difference and final element methods).
4. Algebra and analysis – typical for 1st level of engineering studies.

Recommended (yet, not necessary) – elementary knowledge on PDEs and basics of functional analysis.

Learning outcomes

Knowledge:

EW1 – Student knows principles of formulation of boundary-value problems for incompressible flows, the concept of a weak formulation and the repertoire of admissible boundary conditions (essential and natural).
EW2 – Student knowns general principles of formulation optimization problems for PDEs
EW3 – Student knows necessary conditions for extrema of functionals formulated for solutions of thermomechanical equations.
EW4 – Student understand the need and principle of application of the adjoint technique to determinations of the functional's gradient.
EW5 – Student understands basics of the differential shape calculus and its applications to various shape optimization problems.

Skills:

EU1 – Student can provide/derive a weak formulation of a given boundary problem in thermomechanics.

EU2 – In simple cases, student is able to formulate the Lagrange functional and derive strong and weak forms of the adjoint problem, including appropriate boundary conditions.
EU3 – In simple cases, student is able to calculate the shape derivative of a given functional using formulas from the shape calculus
EU4 – Student is able – working alone or in a team – to develop his own numerical code solving primal and adjoint equations in a simple optimal flow control problem.
EU5 – Student is able to apply standard gradient-based optimization algorithms to solve a simple optimization problem in fluid mechanics.

Social skills:

ES1 – Student can work on a team and is aware of responsibility for correct and timely execution of an assigned task.

Content of the course:

Lectures	Hours
Introduction: overview of fundamental concepts, definitions and theorems from algebra, optimization, differential equations and functional analysis.	4
General concept of gradient-based unconstrained and constrained optimization with a multidimensional control space, tangent-approximation versus the adjoint approach on the example of finite-dimensional nonlinear algebraic problem, adjoint approach to optimal control of a dynamical system with discrete and continuous time.	4
Optimal control of a solution of a partial differential equation – simple examples	2
Case study – optimal control of a mixing process via tangent motion of a flow domain boundary with different goal functionals.	4
Case study – optimal mixing realized via time-dependent volumetric force.	2
Case study – minimization of pressure losses in a wall-confined flow past a rotating obstacle.	2
Case study – problem of unsteady flow in a pipe system with multiply inlets and outlets formulated as the optimal flow control problem	2
Case study – optimal dosing problem for passive scalar transported in a viscous incompressible medium	2
Case study – optimal control of volumetric flow rates in a branched-pipe system using least-square formulation and nonlinear inlet/outlet conditions.	2
Foundations of differential and integral calculus of surfaces (tangent differential operators, integral theorems), foundations of shape calculus: the concepts of shape derivative of a functions and functional, the Hadamard formula.	3
Variational approach to shape derivative of a functional, application of the adjoint operator to avoid explicit evaluation of the shape derivative of primal and adjoint solutions of the state equations. Examples.	2
Case study – determination of shape of an obstacle which minimizes the pressure losses in a wall-confined flow.	1
Optimal flow control by programmed deformation of the flow domain. Non-cylindrical shape calculus and differentiations of a functional with respect to change of shape of the domain in time. Example.	3
Case study – optimal control of flapping motion for maximization of the lift force (Lattice Boltzmann based simulations)	2

Summary, perspectives and suggestion of other applications, further reading	1
Project (team project) On the bases of presented cases studies, student are expected to prepare home projects (realized in small teams, 2-3 students each) which will consist in development the own numerical codes for primal and adjoint problems, determination of the goal functional's gradient and finding optimal solution. In these projects, student will be allowed (and encouraged) to use available libraries of numerical tools and/or noncommercial packages dedicated to PDE-based problems (e.g., PDE MATLAB Toolbox z Matlaba, CFD Toolbox from Quickersim, package Deal.II , FEniCS or Nektar++). The final outcome also will include a full project report and presentation during the reporting seminar.	

Literature and other materials

1. Presentations prepared and provided by a lecturer
2. Manuals and tutorial for chosen programming and computational tools
3. Scientific papers suggested by a lecturer (additional reading)
4. Alfio Quarteroni, Numerical Models for Differential Problems. Series: Modeling, Simulation and Applications, vol. 8. 3rd Ed., Springer, 2017.
5. Andrea Manzoni, Alfio Quarteroni, Sandro Salsa, Optimal Control of Partial Differential Equations. Analysis, Approximation and Applications. Applied Mathematical Sciences, vol. 207. Springer Nature Switzerland, 2021.
6. Hideyuki Azegami, Shape Optimization Problems. Springer Optimization and Its Applications, vol. 164. Springer Nature Singapore, 2020

Student's workload

Form of activity	Hours (average)
Contact hours (lectures)	36
Contact hours (consulting)	4
Self-study (papers, book chapters), home project	50
Total	90

Didactic tools:

1. Lecture presentations (PDF, Power Point)
2. Instructions for home projects
3. Homeworks
4. Websites of used numerical tools and programming environments

Evaluation of student's work and progress

Evaluated elements of student's work include: activity in realization of assigned home tasks, involvement and contribution to realizations of the home project and quality of the final report, presentation of the results during the reporting seminar.

Realization of learning outcomes

LO	Relation to the whole programme	EO	Didactic tools	Evaluation
EW1		C1,C2	Lecture, self-study	Mark in the range 2-5
EW2		C2, C3	Lecture, self-study	As above
EW3		C2,C3,C4	Lecture, self-study	As above
EW4		C3,C4,C5	Lecture, self-study	As above
EW5		C3,C4,C5	Lecture, self-study	As above
EU1		C1	Self-study, homework	As above
EU2		C2,C3	Self-study, homework, contribution to the project	As above
EU3		C2,C3,C4	Self-study, homework, contribution to the project	As above
EU4		C3,C4,C5,C6	Self-study, homework, contribution to the project	As above
EU5		C3,C4,C5,C6	Self-study, homework, contribution to the project	As above
ES1		C6	Contribution to the project	As above